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Full Length Research Article

GENERAL LINEAR ACCELERATION VECTOR BASED ON THE GOLDEN METRIC TENSOR IN SPHERICAL POLAR COORDINATES (PAPER I)

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ABSTRACT

In this paper we derived linear acceleration vector using golden metric tensor in spherical polar coordinates. Coefficients of affine connection were also evaluated based upon golden metric tensor.

Key words: Golden Metric Tensor, Geodesic Equation, Coefficients of Affine Connection.

INTRODUCTION

The properties of geodesics differ from those of straight lines. For example, on a plane, parallel lines never meet, but this is not so far geodesics on the surface of the earth. For example, lines of longitude are parallel at the equator, but intersect at the poles. Analogously, the world lines of test particles in free fall are space time geodesics, the straightest possible lines in space time. But still there are crucial differences between them and the truly straight lines that can be traced out in the gravity-free space time of special relativity (Gnedin, Nickolay, 2005). Einstein's equations are the centre piece of general relativity. They provide a precise formulation of the relationship between space time geometry and the properties of matter, using the language of mathematics. More concretely, they are formulated using the concepts of Riemannian geometry, in which the geometric properties of a space (or a space time) are described by a quantity called a metric. The metric encodes the information needed to compute the fundamental geometric notions of distance and angle in a curved space (or space time) (Hogan, Craig). The metric function and its rate of change from point to point can be defining a geometrical quantity called the Riemann curvature tensor, which describes exactly how the space (or space time) is curved at each point. In general relativity, the metric and the Riemann curvature tensor are quantities define at each point in space time. It is based on the above argument explanation. Howusu introduced, by postulation, a second natural and satisfactorily generalization or extension of the schwarzschild's metric tensor from the gravitational fields of all static homogeneous spherical distribution of mass to the gravitational fields of all spherical distributions of mass-named as the golden metric tensor for all gravitational fields in nature (Howusu, 1991)

Golden metric tensor

In this paper we introduced golden metric tensor for all gravitational fields in nature as follows (Howusu, 2009; Howusu , 2009; Howusu, 2010). The covariant form of all golden metric tensor for all gravitational fields in nature as:

$$g_{11} = \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1} \tag{1}$$

$$g_{22} = r^2 \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1} \tag{2}$$

$$g_{33} = r^2 \sin^2\theta \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1} \tag{3}$$

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$$g_{00} = -\left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}$$

(4)

$$g_{uv} = 0, \text{otherwise}$$

(5)

Where f is the gravitational scalar potential of the space time, the golden metric tensor and also contains the following physical effects:

- Gravitational space contraction
- Gravitational time dilation
- Gravitational polar angle contraction
- Gravitational azimuthal angle contraction

While the contravariant form of golden metric tensor given as:

$$g^{11} = \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}$$ (6)

$$g^{22} = r^2 \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}$$ (7)

$$g^{33} = r^2 \sin^2\theta \left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}$$ (8)

$$g^{00} = -\left\{1 + \frac{2}{c^2} f(r, \theta, \phi, x^0)\right\}^{-1}$$

(9)

$$g^{uv} = 0, \text{otherwise}$$

(10)

Formulation of the general dynamical laws of gravitation

It is well known that all of Newton's dynamical laws of gravitation are founded on the experimental physical facts available in his day. The instantaneous active mass passive mass and the inertial mass of a particle of non-zero rest mass are given by:

$$m_A = m_p = m_I = m_0$$ (11)

In all proper inertial reference frames and proper times.

This statement may be called Newton's principle of mass. According to the classic scientific method introduced by G. Galileo (the father of mechanics) and Newton the natural laws of mechanics are determined by experimental physical facts. Therefore today's experimental revisions of the definitions of inertial, passive and active masses of a particle on non-zero rest mass induce a corresponding revision of Newton's dynamical laws of gravitation which are now formulated (Howusu, 2011).

Super General Planetary Equation are given by (Howusu , 2012; Rajput , 2010; Spiegel , 1974)

$$(m_I)_H = (m_p)_H = \left[1 - \frac{u^2}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1}\right]^{-\frac{1}{2}} \cdot \left(1 + \frac{2}{c^2} f\right)^{\frac{1}{2}} m_0$$ (12)

As given by equation (11) above

$\underline{g}_H$ = Riemannian acceleration due to gravity

$\underline{u}_H$ = Riemannian velocity vector

But force is defined as the rate of change of momentum

$$\frac{d}{d\tau}\left\{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}\right)^{-1}\right]^{-\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0\underline{u}_H\right\}$$

$$=\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{-\frac{1}{2}}.\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0\underline{g}_H \tag{13}$$

Where $\left(1+\frac{2}{c^2}f\right)$ is Riemannian factor

Equivalently,

$$\frac{d}{d\tau}\{(m_I)_H\,.\underline{u}_H\} = (m_p)_H\underline{g}_H \tag{14}$$

$$(m_I)_H\,\underline{a}_H + \left\{\frac{d}{d\tau}[(m_I)_H]\right\}\underline{u}_H = (m_p)_H\underline{g}_H \tag{15}$$

$$\underline{a}_H + \frac{1}{(m_I)_H}\left\{\frac{d}{d\tau}[(m_I)_H]\right\}\underline{u}_H = \underline{g}_H \tag{16}$$

$\underline{a}_H \equiv$ general acceleration vector.

Equation (16) above referred to as super general geodesics equation of motion.
Velocity Tensor

$$\dot{x}^\mu = \{\dot{x}^1, \dot{x}^2, \dot{x}^3, \dot{x}^0\}$$

$$x^1 = r,\ \ x^2 = \theta,\ \ x^3 = \phi,\ \ x^0 = ct \tag{17}$$

$$\dot{x}^\mu = \{\dot{r}, \dot{\theta}, \dot{\phi}, c\dot{t}\} \tag{18}$$

Velocity vector

$$\underline{u}_H = \left(\sqrt{g_{11}}\dot{x}^1, \sqrt{g_{22}}\dot{x}^2, \sqrt{g_{33}}\dot{x}^3, \sqrt{g_{00}}\dot{x}^0\right) \tag{19}$$

Based upon golden metric tensor

$$u_r = \sqrt{g_{11}}\dot{r} = \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{r} \tag{20}$$

$$u_\theta = \sqrt{g_{22}}\dot{\theta} = r\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{\theta} \tag{21}$$

$$u_\varphi = \sqrt{g_{33}}\dot{\phi} = r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{\phi} \tag{22}$$

$$u_0 = \sqrt{g_{00}}\dot{x}^0 = \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} ic\dot{t} \tag{23}$$

Where we have make use of Golden metric tensor in equations (11)-(14)

Acceleration Tensor

Acceleration tensors define by geodesics equation as:

$$a^{\mu} = \ddot{x}^{\mu} + \Gamma^{\mu}_{\alpha\beta}\dot{x}^{\alpha}\dot{x}^{\beta} \tag{24}$$

Where $\Gamma^{\mu}_{\alpha\beta}$ is defined as coefficient of affine connection gives as:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}g^{\mu\varepsilon}\left(g_{\alpha\varepsilon,\beta} + g_{\varepsilon\beta,\alpha} - g_{\alpha\beta,\varepsilon}\right) \tag{25}$$

Putting μ=1

$$a^{1} = \ddot{r} + \Gamma^{1}_{11}(\dot{r})^{2} + \Gamma^{1}_{22}(\dot{\theta})^{2} + \Gamma^{1}_{33}(\dot{\phi})^{2} \tag{26}$$

Putting μ=2

$$a^{2} = \ddot{\theta}^{2} + 2\Gamma^{2}_{12}(\dot{r}\dot{\theta}) + \Gamma^{2}_{33}(\dot{\phi})^{2} \tag{27}$$

Putting μ=3

$$a^{3} = \ddot{\phi}^{2} + 2\Gamma^{3}_{13}(\dot{r}\dot{\phi}) + 2\Gamma^{3}_{23}(\dot{\theta}\dot{\phi}) \tag{28}$$

Putting μ=0

$$a^{0} = c\ddot{t} + 2c\Gamma^{0}_{01}(\dot{t}\dot{r}) \tag{29}$$

Using

$$x^{0} = ct$$

By employing equation (25) above, the evaluated coefficients of affine connection gives:

$$\Gamma^{1}_{00} = \frac{1}{c^{2}}\left(1 + \frac{2}{c^{2}}f\right)f_{,1} \tag{30}$$

$$\Gamma^{1}_{01} = \Gamma^{1}_{10} = -\frac{1}{c^{2}}\left(1 + \frac{2}{c^{2}}f\right)^{-1} f_{,0} \tag{31}$$

$$\Gamma^{1}_{11} = -\frac{1}{c^{2}}\left(1 + \frac{1}{c^{2}}f\right)^{-1} f_{,1} \tag{32}$$

$$\Gamma^{1}_{12} = \Gamma^{1}_{21} = -\frac{1}{c^{2}}\left(1 + \frac{2}{c^{2}}f\right)^{-1} f_{,2} \tag{33}$$

$$\Gamma^{1}_{13} = \Gamma^{1}_{31} = -\frac{1}{c^{2}}\left(1 + \frac{2}{c^{2}}f\right)^{-1} f_{,3} \tag{34}$$

$$\Gamma^{1}_{22} = -\frac{r^{2}}{c^{2}}\left(1 + \frac{2}{c^{2}}f\right)^{-1} f_{,1} - r \tag{35}$$

$$\Gamma^{1}_{33} = \frac{r^{2}\sin^{2}\theta}{c^{2}}\left(1 + \frac{2}{c^{2}}f\right)^{-1} f_{,1} - r\sin^{2}\theta \tag{36}$$

and

$$\Gamma^2_{00} = \frac{1}{c^2r^2}\left(1+\frac{2}{c^2}f\right)f_{,2} \tag{37}$$

$$\Gamma^2_{02} = \Gamma^2_{20} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,0} \tag{38}$$

$$\Gamma^2_{11} = \frac{1}{c^2r^2}\left(1+\frac{1}{c^2}f\right)^{-1}f_{,2} \tag{39}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \tag{40}$$

$$\Gamma^2_{22} = -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \tag{41}$$

$$\Gamma^2_{23} = \Gamma^2_{32} = -\frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,3} \tag{42}$$

$$\Gamma^2_{33} = \frac{\sin^2\theta}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} - \sin\theta\cos\theta \tag{43}$$

and

$$\Gamma^3_{00} = \frac{1}{c^2r^2\sin^2\theta}\left(1+\frac{2}{c^2}f\right)f_{,3} \tag{44}$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,3} \tag{45}$$

$$\Gamma^3_{11} = \frac{1}{c^2r^2\sin^2\theta}\left(1+\frac{2}{c^2}f\right)f_{,3} \tag{46}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1} \tag{47}$$

$$\Gamma^3_{22} = \frac{1}{c^2\sin^2\theta}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,3} \tag{48}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \tag{49}$$

$$\Gamma^3_{33} = -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,3} \tag{50}$$

And

$$\Gamma^0_{00} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,0} \tag{51}$$

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1} \tag{52}$$

$$\Gamma^0_{02} = \Gamma^0_{20} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \tag{53}$$

$$\Gamma^0_{03} = \Gamma^0_{30} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3} \tag{54}$$

$$\Gamma^0_{01} = -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,0} \tag{55}$$

$$\Gamma^0_{22} = -\frac{r^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,0} \tag{56}$$

$$\Gamma^0_{33} = -\frac{r^2\sin^2\theta}{c^2}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,0} \tag{57}$$

$$\Gamma^\mu_{\alpha\beta} = 0; \quad \text{otherwise} \tag{58}$$

Hence the acceleration vector

$$\underline{a}_H\left(\sqrt{g_{11}}a^1, \sqrt{g_{22}}a^2, \sqrt{g_{33}}a^3, \sqrt{g_{00}}a^0\right) \tag{59}$$

By putting the results of coefficients of affine connection, covariant form of golden metric tensors into equation (59), we obtained acceleration vector equations as:

$$a_r = \sqrt{g_{11}}a^1$$

$$= \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} a^1$$

$$= \left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\begin{matrix}\ddot{r} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}(\dot{r})^2 - r\left(1+\frac{2}{c^2}f\right)(\dot{\theta})^2 \\ -r\sin^2\theta\left(1+\frac{2}{c^2}f\right)(\dot{\phi})^2\end{matrix}\right] \tag{60}$$

$$a_\theta = \sqrt{g_{22}}a^2$$

$$= r\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} a^2$$

$$= r\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\ddot{\theta} + \frac{2}{r}(\dot{r}\dot{\theta}) - \sin\theta\cos\theta(\dot{\phi})^2\right] \tag{61}$$

$$a_\phi = \sqrt{g_{33}}a^3$$

$$= r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\ddot{\phi} + \frac{2}{r}(\dot{r}\dot{\theta}) + 2\cot\theta(\dot{\theta}\dot{\phi})\right] \tag{62}$$

$$a_0 = \sqrt{g_{00}}a^0$$

$$= \left(1+\frac{2}{c^2}f\right)i\left[c\ddot{t} + \frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1} f_{,1}\right] \tag{63}$$

Hence equations (60), (61), (62) and (63) are linear acceleration vector equations based upon golden metric tensor.

Conclusion and Results

In this paper we have succeeded in obtaining (20) - (23) which referred to as velocity vector equations based upon the golden metric tensor. We further obtained equations (60) – (63) which referred to as linear acceleration vector equations. These results obtained in this paper are now available for both physics and mathematician alike to apply them in solving planetary problems based upon Riemannian geometry.

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