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RESEARCH ARTICLE

LEAP HYPER-ZAGREB INDICES AND THEIR POLYNOMIALS OF CERTAIN GRAPHS

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ABSTRACT

We introduce the leap hyper-Zagreb indices of a graph. In this paper, we compute the leap hyper-Zagreb indices and their polynomials of wheel, gear, helm, flower and sunflower graphs.

Key words: leap hyper Zagreb indices, leap hyper Zagreb polynomials, wheel graphs.

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INTRODUCTION

Let G be a simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree of a vertex v is the number of edges incident to v and is denoted by $d(v)$. The distance between two vertices u and v of a graph G is the number of edges in a shortest path connecting them and it is denoted by $d(u, v)$. For a vertex v in G, the open neighborhood of v is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$, where k is a positive integer. The k-distance degree, denoted by $d_k(v)$, of a vertex $v \in V(G)$ is the number of k neighbors of v in G, see (1). We refer to (2) for undefined terminology and notation not given here.

The second leap Zagreb index was introduce by Naji *et al*. in (1) and defined as

$$LM_2(G) = \sum_{uv \in E(G)} d_2(u) d_2(v)$$

Recently, some new leap indices were introduced and studied such as sum connectivity leap and geometric-arthimetic leap indices (3), F-leap indices (4), augmented leap index (5) and minus leap and square leap indices (6).

A new version of the first leap Zagreb index is defined as

$$LM_1^*(G) = \sum_{uv \in E(G)} \left[d_2(u) + d_2(v)\right].$$

We now define the first and second leap hyper-Zagreb indices as

$$HLM_1(G) = \sum_{uv \in E(G)} \left[d_2(u) + d_2(v)\right]^2$$

$$HLM_2(G) = \sum_{uv \in E(G)} \left[d_2(u) d_2(v)\right]^2.$$

In (7), Shirdel *et al*. introduced the hyper-Zagreb index. In recent years, some new hyper-Zagreb type indices were introduced and studied such as hyper Revan indices (8), reverse hyper-Zegreb indices (9), multiplicative hyper-Zagreb indices (10), K hyper Banhatti indices (11).

We introduce the general first and second leap Zagreb indices defined as

$$LM_1^a(G) = \sum_{uv \in E(G)} \left[d_2(u) + d_2(v)\right]^a. \qquad \text{..........(1)}$$

$$LM_2^a(G) = \sum_{uv \in E(G)} \left[d_2(u) d_2(v)\right]^a. \qquad \text{..........(2)}$$

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Considering the leap Zagreb indices, we propose the first and second leap Zagreb polynomials and the first and second leap hyper-Zagreb polynomials of G, defined respectively as

$$LM_1^*(G,x) = \sum_{uv\in E(G)} x^{[d_2(u)+d_2(v)]} \qquad \text{………………………..(3)}$$

$$LM_2(G,x) = \sum_{uv\in E(G)} x^{d_2(u)d_2(v)} \qquad \text{………………………..(4)}$$

$$HLM_1(G,x) = \sum_{uv\in E(G)} x^{[d_2(u)+d_2(v)]^2} \qquad \text{………………………..(5)}$$

$$HLM_2(G,x) = \sum_{uv\in E(G)} x^{[d_2(u)d_2(v)]^2}. \qquad \text{………………………..(6)}$$

We consider wheel graphs and wheel type graphs, see (12). In this paper, the leap hyper-Zagreb indices and their polynomials, and general first and second leap Zagreb indices of wheels, gear graphs, helm graphs flower graphs, sunflower graphs are determined.

2. Wheels W_{n+1}

The wheel W_{n+1} is defined to be the join of cycle C_n and complete graph K_1. Let $G = W_{n+1}$. The graph G has $n+1$ vertices and $2n$ edges. The vertex of K_1 is called apex and the vertices of C_n are called rim vertices.

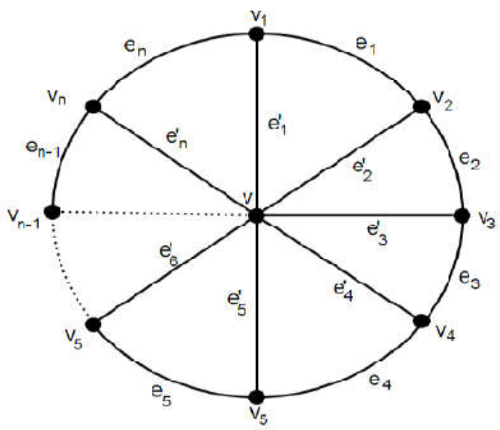

Figure 1. Graph W_{n+1}

There are two types of the 2-distance degree of edges in W_{n+1} as given in Table 1.

Table 1.

$d_2(u), d_2(v)\backslash uv \in E(G)$	$(0, n-3)$	$(n-3, n-3)$
Number of edges	n	n

Theorem 1. Let $G=W_{n+1}$ be a wheel with $n+1$ vertices and $2n$ edges, $n \geq 3$. Then

a) $LM_1^a(G) = (1+2^a)n(n-3)^a$. ……………………….(7)

b) $LM_2^a(G) = n(n-3)^{2a}$. ……………………….(8)

Proof: (a) By using equation (1) and Table 1, we have

$$LM_1^a(G) = \sum_{uv\in E(G)} [d_2(u)+d_2(v)]^a$$

$$= n(0+n-3)^a + n(n-3+n-3)^a = (1+2^a)n(n-3)^a.$$

(b) By using equation (2) and Table 1, we have

$$LM_2^a(G) = \sum_{uv\in E(G)} [d_2(u)d_2(v)]^a$$

$$= n(0\times n-3)^a + n[(n-3)(n-3)]^a = n(n-3)^{2a}.$$

Corollary 1.1. Let W_{n+1} be a wheel, $n \geq 3$. Then

(a) $LM_1^*(W_{n+1}) = 3n(n-3)$.

(b) $HLM_1(W_{n+1}) = 5n(n-3)^2$.

Proof: Put a = 1, 2 in equation (7), we obtain the above results.

Corollary 1.2. Let W_{n+1} be a wheel, $n \geq 3$. Then

(a) $LM_2(W_{n+1}) = n(n-3)^2$, see (12).

(b) $HLM_2(W_{n+1}) = n(n-3)^4$.

Proof: Put a = 1, 2 in equation (8), we get the above results.

Theorem 2. Let $G=W_{n+1}$ be a wheel, $n \geq 3$. Then

a) $LM_1^*(W_{n+1}, x) = nx^{n-3} + nx^{2(n-3)}$.

b) $LM_2(W_{n+1}, x) = nx^0 + nx^{(n-3)^2}$

c) $HLM_1(W_{n+1}, x) = nx^{(n-3)^2} + nx^{4(n-3)^2}$.

d) $HLM_2(W_{n+1}, x) = nx^0 + nx^{(n-3)^4}$.

Proof: (a) From equation (3) and Table 1, we have

$$LM_1^*(W_{n+1}, x) = \sum_{uv \in E(G)} x^{d_2(u)+d_2(v)} = nx^{0+n-3} + nx^{n-3+n-3} = nx^{n-3} + nx^{2(n-3)}$$

(b) From equation (4) and Table 1, we get

$$LM_2(W_{n+1}, x) = \sum_{uv \in E(G)} x^{d_2(u)d_2(v)} = nx^{0(n-3)} + nx^{(n-3)(n-3)} = nx^0 + nx^{(n-3)^2}.$$

(c) From equation (5) and Table 1, we obtain

$$HLM_1(W_{n+1}, x) = \sum_{uv \in E(G)} x^{[d_2(u)+d_2(v)]^2} = nx^{(0+n-3)^2} + nx^{(n-3+n-3)^2} = nx^{(n-3)^2} + nx^{4(n-3)^2}.$$

(d) By using equation (6) and Table 1, we establish

$$HLM_2(W_{n+1}, x) = \sum_{uv \in E(G)} x^{[d_2(u)d_2(v)]^2} = nx^{[0\times(n-3)]^2} + nx^{[(n-3)(n-3)]^2} = nx^0 + nx^{(n-3)^4}.$$

3. Gear Graphs G_n

The gear graph G_n is a graph obtained from wheel W_{n+1} by adding a vertex between each pair of adjacent rim vertices. Clearly G_n has $2n$+1 vertices and $3n$ edges.

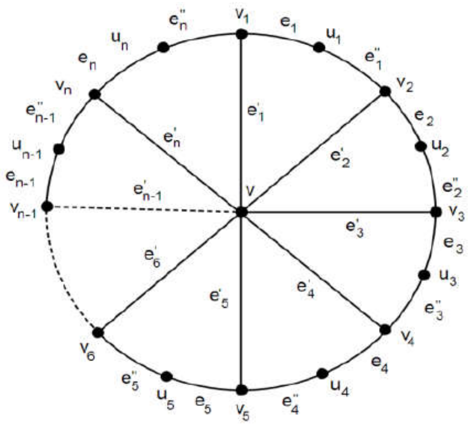

Figure 2. Graph G_n

In G_n, there are two types of the 2-distance degree of edges as given in Table 2.

Table 2.

$d_2(u), d_2(v) \backslash uv \in E(G_n)$	$(n, n-1)$	$(3, n-1)$
Number of edges	n	$2n$

Theorem 3. Let G_n be a gear graph with $3n$ edges. Then

a) $LM_1^a\left(G_n\right)=n\left(2n-1\right)^a+2n\left(n+2\right)^a$. ……………………………………..(9)

b) $LM_2^a\left(G_n\right)=n\left[n\left(n-1\right)\right]^a+2n\left[3\left(n-1\right)\right]^a$. ………………………………….(10)

Proof: (a) By using equation (1) and Table 2, we obtain

$$LM_1^a\left(G_n\right)=\sum_{uv\in E(G_n)}\left[d_2\left(u\right)+d_2\left(v\right)\right]^a$$
$$=n\left(n+n-1\right)^a+2n\left(3+n-1\right)^a$$
$$=n\left(2n-1\right)^a+2n\left(n+2\right)^a.$$

(b) By using equation (2) and Table 2, we obtain

$$LM_2^a\left(G_n\right)=\sum_{uv\in E(G_n)}\left[d_2\left(u\right)d_2\left(v\right)\right]^a$$
$$=n\left[n\left(n-1\right)\right]^a+2n\left[3\left(n-1\right)\right]^a.$$

Corollary 3.1. Let G_n be a gear graph with $3n$ edges. Then

(a) $LM_1^*\left(G_n\right)=4n^2+3n.$

(b) $HLM_1\left(G_n\right)=6n^2+4n^2+9n.$

Proof: Put a = 1, 2 in equation (9), we get the above desired results.

Corollary 3.2. Let G_n be a gear graph with $3n$ edges. Then

(a) $LM_2\left(G_n\right)=n^3+5n^2-6n,$ see (12).

(b) $HLM_2\left(G_n\right)=n^5-2n^4+19n^3-36n^2+18n.$

Proof: Put a = 1, 2 in equation (10), we get the above desired results.

Theorem 4. Let G_n be a gear graph with $3n$ edges. Then

a) $LM_1^*\left(G_n,x\right)=nx^{2n-1}+2nx^{n+2}.$

b) $LM_2\left(G_n,x\right)=nx^{n(n-1)}+2nx^{3(n-1)}.$

c) $HLM_1\left(G_n,x\right)=nx^{(2n-1)^2}+2nx^{(n+2)^2}.$

d) $HLM_2\left(G_n,x\right)=nx^{n^2(n-1)^2}+2nx^{9(n-1)^2}.$

Proof: (a) From equation (3) and Table 2, we derive

$$LM_1^*\left(G_n,x\right)=\sum_{uv\in E(G_n)}x^{\left[d_2(u)+d_2(v)\right]}=nx^{n+n-1}+2nx^{3+n-1}=nx^{2n-1}+2nx^{n+2}.$$

(b) From equation (4) and Table 2, we establish

$$LM_2\left(G_n,x\right)=\sum_{uv\in E(G_n)}x^{d_2(u)d_2(v)}=nx^{n(n-1)}+2n^{3(n-1)}.$$

(c) From equation (5) and Table 2, we obtain

$$HLM_1\left(G_n,x\right)=\sum_{uv\in E(G_n)}x^{\left[d_2(u)+d_2(v)\right]^2}=nx^{(n+n-1)^2}+2nx^{(3+n-1)^2}=nx^{(2n-1)^2}+2nx^{(n+2)^2}.$$

(d) By using equation (6) and Table 2, we have

$$HLM_2\left(G_n,x\right)=\sum_{uv\in E(G_n)}x^{\left[d_2(u)d_2(v)\right]^2}=nx^{[n(n-1)]^2}+2nx^{[3(n-1)]^2}=nx^{n^2(n-1)^2}+2nx^{9(n-1)^2}.$$

4. Helm Graphs H_n

A helm graph H_n is a graph which is obtained from W_{n+1} by attaching a pendant edge to each rim vertex. Clearly H_n has $2n$+1 vertices and $3n$ edges.

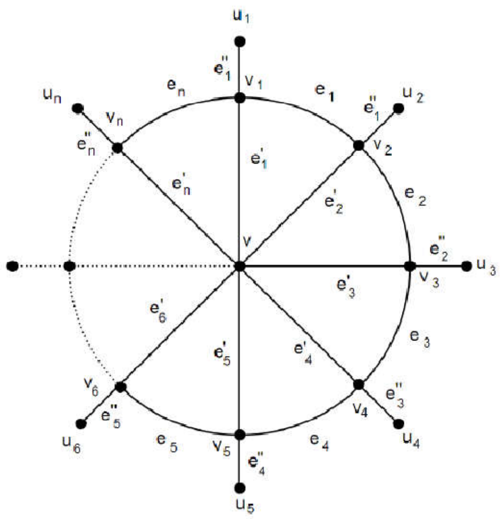

Figure 3. Graph H_n

In H_n, there are three types of the 2-distance degree of edges as given in Table 3.

Table 3.

$d_2(u), d_2(v) \backslash uv \in E(H_n)$	$(n, n-1)$	$(3, n-1)$	$(n-1, n-1)$
Number of edges	n	n	n

Theorem 5. Let H_n be a helm graph with $3n$ edges. Then

a) $LM_1^a(H_n) = n(2n-1)^a + n(n+2)^a + n(2n-2)^a$. ……………………………….(11)

b) $LM_2^a(H_n) = n[n(n-1)]^a + n[3(n-1)]^a + n(n-1)^{2a}$. ………………………………..(12)

Proof: (a) By using equation (1) and Table 3, we deduce

$$LM_1^a(H_n) = \sum_{uv \in E(H_n)} [d_2(u) + d_2(v)]^a$$

$$= n(n + n - 1)^a + n(3 + n - 1)^a + n(n - 1 + n - 1)^a$$

$$= n(2n - 1)^a + n(n + 2)^a + n(2n - 2)^a.$$

(b) By using equation (2) and Table 3, we deduce

$$LM_2^a(H_n) = \sum_{uv \in E(H_n)} [d_2(u) d_2(v)]^a$$

$$= n[n(n - 1)]^a + n[3(n - 1)]^a + n[(n - 1)(n - 1)]^a$$

$$= n[n(n - 1)]^a + n[3(n - 1)]^a + n(n - 1)^{2a}.$$

Corollary 5.1. Let H_n be a helm graph with $3n$ edges. Then

(a) $LM_1^*(H_n) = 5n^2 - n.$

(b) $HLM_1(H_n) = 9n^3 - 8n^2 + 9n.$

Proof: Put $a = 1, 2$ in equation (11), we get the above desired results.

Corollary 5.2. Let H_n be a helm graph with, $3n$ edges. Then

(a) $LM_2(H_n) = 2n^3 - 2n,$ see (12).

(b) $HLM_2(H_n) = (n - 1)^2 (2n^3 - 2n^2 + 10n).$

Proof: Put $a = 1, 2$ in equation (12), we obtain the above desired results.

Theorem 6. Let H_n be a helm graph with $3n$ edges. Then

a) $LM_1^*(H_n, x) = nx^{2n-1} + nx^{n+2} + nx^{2n-2}$.

b) $LM_2(H_n, x) = nx^{n(n-1)} + nx^{3(n-1)} + nx^{(n-1)^2}$.

c) $HLM_1(H_n, x) = nx^{(2n-1)^2} + nx^{(n+2)^2} + nx^{(2n-2)^2}$.

d) $HLM_2(H_n, x) = nx^{[n(n-1)]^2} + nx^{[3(n-1)]^2} + nx^{[(n-1)]^4}$.

Proof: (a) From equation (3) and Table 3, we derive

$$LM_1^*(H_n, x) = \sum_{uv \in E(H_n)} x^{d_2(u)+d_2(v)} = nx^{n+n-1} + nx^{3+n-1} + nx^{n-1+n-1}$$

$$= nx^{2n-1} + nx^{n+2} + nx^{2n-2}.$$

(b) From equation (4) and Table 3, we deduce

$$LM_2(H_n, x) = \sum_{uv \in E(H_n)} x^{d_2(u)d_2(v)} = nx^{n(n-1)} + nx^{3(n-1)} + nx^{(n-1)(n-1)}$$

$$= nx^{n(n-1)} + nx^{3(n-1)} + nx^{(n-1)^2}.$$

(c) From equation (5) and Table 3, we obtain

$$HLM_1(H_n, x) = \sum_{uv \in E(H_n)} x^{[d_2(u)+d_2(v)]^2} = nx^{[n+(n-1)]^2} + nx^{[3+(n-1)]^2} nx^{[(n-1)+(n-1)]^2}$$

$$= nx^{(2n-1)^2} + nx^{(n+2)^2} + nx^{(2n-2)^2}.$$

(d) By using equation (6) and Table 3, we have

$$HLM_2(H_n, x) = \sum_{uv \in E(H_n)} x^{[d_2(u)d_2(v)]^2} = nx^{[n(n-1)]^2} + nx^{[3(n-1)]^2} + nx^{[(n-1)(n-1)]^2}$$

$$= nx^{[n(n-1)]^2} + nx^{[3(n-1)]^2} + nx^{[(n-1)(n-1)]^2}.$$

5. Flower Graphs Fl_n

A graph Fl_n is a flower graph which is obtained from a helm graph by joining each pendant vertex to the apex of the helm graph. Clearly Fl_n has $2n+1$ vertices and $4n$ edges.

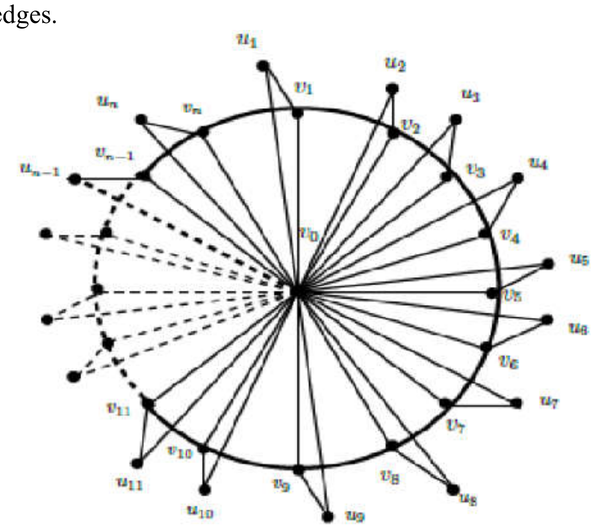

Figure 4. Graph Fl_n

In Fl_n, there are 4 types of the 2-distance degree of edges as given in Table 4.

Table 4.

$d_2(u), d_2(v) \backslash uv \in E(Fl_n)$	$(0, n-5)$	$(0, n-2)$	$(n-5, n-2)$	$(n-5, n-5)$
Number of edges	n	n	n	n

Theorem 7. Let $G = Fl_n$ be a flower graph with $2n+1$ vertices and $4n$ edges. Then

a) $LM_1^a(Fl_n) = 3n(n-5)^a + n(n-2)^a + n(2n-7)^a + n(2n-10)^a$. ……………………………..(13)

b) $LM_2^a(Fl_n) = n[(n-5)(n-2)]^a + n(n-5)^{2a}$. ……………………………(14)

Proof: (a) By using equation (1) and Table 4, we have

$$LM_1^a(Fl_n) = \sum_{uv\in E(G)} [d_2(u)+d_2(v)]^a$$
$$= n(0+n-5)^a + n(0+n-2)^a + n(n-5+n-2)^a + n(n-5+n-5)^a$$
$$= 3n(n-5)^a + n(n-2)^a + n(2n-7)^a + n(2n-10)^a.$$

(b) By using equation (2) and Table 4, we obtain

$$LM_2^a(Fl_n) = \sum_{uv\in E(G)} [d_2(u)d_2(v)]^a$$
$$= n[0\times(n-5)]^a + n[0\times(n-2)]^a + n[(n-5)(n-2)]^a + n[(n-5)(n-5)]^a$$
$$= n[(n-5)(n-2)]^a + n(n-5)^{2a}.$$

Corollary 7.1. Let Fl_n be a flower graph with $4n$ edges. Then

(a) $LM_1^*(Fl_n) = 8n^2 - 34n.$
(b) $LM_1(Fl_n) = 8n^3 - 62n^2 + 128n.$

Proof: Put $a = 1, 2$ in equation (13), we obtain the above desired results.

Corollary 7.2. Let Fl_n be a flower graph with $4n$ edges. Then

(a) $LM_2(Fl_n) = 2n^3 - 17n^2 + 35n$, see (12).
(b) $HLM_2(Fl_n) = 2n^5 - 34n^4 + 219n^3 - 415n^2 + 725n.$

Proof: Put $a = 1, 2$ in equation (14), we get the above desired results.

Theorem 8. Let $G=Fl_n$ be a flower graph with $4n$ edges. Then

a) $LM_1^*(Fl_n, x) = nx^{n-5} + nx^{n-2} + nx^{2n-7} + nx^{2n-10}.$
b) $LM_2(Fl_n, x) = 2nx^0 + nx^{(n-5)(n-2)} + nx^{(n-5)^2}.$
c) $HLM_1(Fl_n, x) = nx^{(n-5)^2} + nx^{(n-2)^2} + nx^{(2n-7)^2} + nx^{(2n-10)^2}.$
d) $HLM_2(Fl_n, x) = 2nx^0 + nx^{[(n-5)(n-2)]^2} + nx^{[(n-5)]^4}.$

Proof: (a) From equation (3) and Table 4, we obtain

$$LM_1^*(Fl_n, x) = \sum_{uv\in E(G)} x^{[d_2(u)+d_2(v)]} = nx^{0+n-5} + nx^{0+n-2} + nx^{n-5+n-2} + nx^{n-5+n-5}$$
$$= nx^{n-5} + nx^{n-2} + nx^{2n-7} + nx^{2n-10}.$$

(b) From equation (4) and Table 4, we have

$$LM_2(Fl_n, x) = \sum_{uv\in E(G)} x^{d_2(u)d_2(v)} = nx^{0(n-5)} + nx^{0(n-2)} + nx^{(n-5)(n-2)} + nx^{(n-5)(n-5)}$$
$$= 2nx^0 + nx^{(n-5)(n-2)} + nx^{(n-5)^2}.$$

(c) From equation (5) and Table 4, we deduce

$$HLM_1(Fl_n, x) = \sum_{uv\in E(G)} x^{[d_2(u)+d_2(v)]^2} = nx^{(n-5)^2} + nx^{(n-2)^2} + nx^{(2n-7)^2} + nx^{(2n-10)^2}$$

(d) By using equation (6) and Table 4, we derive

$$HLM_2(Fl_n, x) = \sum_{uv\in E(G)} x^{[d_2(u)d_2(v)]^2} = 2nx^0 + nx^{[(n-5)(n-2)]^2} + nx^{(n-5)^4}.$$

6. Sunflower Graphs Sf_n

A graph Sf_n is a sunflower graph which is obtained from the flower graph Fl_n by attaching n pendant edges to the apex vertex. Clearly Sf_n has $3n+1$ vertices and $5n$ edges.

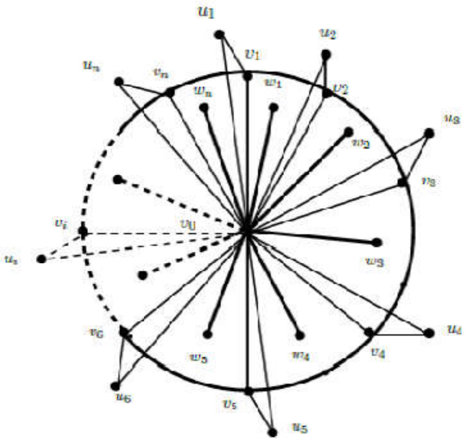

Figure 5. Graph Sf_n

In Sf_n, there are 5 types of the 2-distance degree of edges as given in Table 5.

Table 5.

$d_2(u), d_2(v) \setminus uv \in E(Sf_n)$	$(0, 3n-4)$	$(0, 3n-2)$	$(0, 3n-1)$	$(3n-4, 3n-4)$	$(3n-4, 3n-2)$
Number of edges	n	n	n	n	n

Theorem 9. Let $G = Sf_n$ be a sunflower graph with $3n+1$ vertices and $5n$ edges. Then

a) $LM_1^a(G) = n(3n-4)^a + n(3n-2)^a + n(3n-1)^a + n(6n-8)^a + n(6n-6)^a$. ……..............……..(15)

b) $LM_2^a(G) = n(3n-4)^{2a} + n[(3n-4)(3n-2)]^a$. …………………………..(16)

Proof: (a) By using equation (1) and Table 5, we deduce

$$\begin{aligned} LM_1^a(G) &= \sum_{uv\in E(G)} \left[d_2(u)+d_2(v)\right]^a \\ &= n(0+3n-4)^a + n(0+3n-2)^a + n(0+3n-1)^a + n(3n-4+3n-4)^a + n(3n-4+3n-2)^a \\ &= n(3n-4)^a + n(3n-2)^a + n(3n-1)^a + n(6n-8)^a + n(6n-6)^a. \end{aligned}$$

(b) By using equation (2) and Table 5, we obtain

$$\begin{aligned} LM_2^a(G) &= \sum_{uv\in E(G)} \left[d_2(u)d_2(v)\right]^a \\ &= n[0\times(3n-4)]^a + n[0\times(3n-2)]^a + n[0\times(3n-1)]^a + n[(3n-4)(3n-4)]^a + n[(3n-4)(3n-2)]^a \\ &= n(3n-4)^{2a} + n[(3n-4)(3n-2)]^a. \end{aligned}$$

Corollary 9.1. Let Sf_n be a sunflower graph with $5n$ edges. Then

(a) $LM_1^*(Sf_n) = 21n^2 - 21n.$

(b) $HLM_1(Sf_n) = 99n^3 - 210n^2 + 121n.$

Proof: Put $a = 1, 2$ in equation (15), we get the above desired results.

Corollary 9.2. Let Sf_n be a sunflower graph with $5n$ edges. Then

(a) $LM_2(Sf_n) = 18n^3 - 42n^2 + 24n,$ see (12).

(b) $HLM_2(Sf_n) = n(3n-4)^2(18n^2 - 36n + 20).$

Proof: Put $a = 1, 2$ in equation (16), we obtain the above desired results.

Theorem 10. Let $G=Sf_n$ be a sunflower graph with $5n$ edges. Then

a) $LM_1^*(Sf_n,x)= nx^{3n-4}+nx^{3n-2}+nx^{3n-1}+nx^{6n-8}+nx^{6n-6}$.

b) $LM_2(Sf_n,x)= 3nx^0+nx^{(3n-4)^2}+nx^{(3n-4)(3n-2)}$.

c) $HLM_1(Sf_n,x)= nx^{(3n-4)^2}+nx^{(3n-2)^2}+nx^{(3n-1)^2}+nx^{(6n-8)^2}+nx^{(6n-6)^2}$.

d) $HLM_2(Sf_n,x)= 3nx^0+nx^{(3n-4)^4}+nx^{[(3n-4)(3n-2)]^2}$.

Proof: (a) From equation (3) and Table 5, we have

$$LM_1^*(Sf_n,x)=\sum_{uv\in E(G)} x^{[d_2(u)+d_2(v)]} = nx^{0+3n-4}+nx^{0+3n-2}+nx^{0-3n-1}+nx^{3n-4+3n-4}+nx^{3n-4+3n-2}$$
$$= nx^{3n-4}+nx^{3n-2}+nx^{3n-1}+nx^{6n-8}+nx^{6n-6}.$$

(b) By using equation (4) and Table 5, we derive

$$LM_2(Sf_n,x)=\sum_{uv\in E(G)} x^{d_2(u)d_2(v)} = nx^{0(3n-4)}+nx^{0(3n-2)}+nx^{0(3n-1)}+nx^{(3n-4)(3n-4)}+nx^{(3n-4)(3n-2)}$$
$$=3nx^0+nx^{(3n-4)^2}+nx^{(3n-4)(3n-2)}.$$

(c) From equation (5) and Table 5, we obtain

$$HLM_1(Sf_n,x)=\sum_{uv\in E(G)} x^{[d_2(u)+d_2(v)]^2} = nx^{(3n-4)^2}+nx^{(3n-2)^2}+nx^{(3n-1)^2}+nx^{(6n-8)^2}+nx^{(6n-6)^2}.$$

(d) By using equation (6) and Table 6, we deduce

$$HLM_2(Sf_n,x)=\sum_{uv\in E(G)} x^{[d_2(u)d_2(v)]^2} = 3nx^0+nx^{(3n-4)^4}+nx^{[(3n-4)(3n-2)]^2}.$$

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